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How Does Inside Information Affect Sports Betting Odds?

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bstract

We describe how the presence of insiders with superior information about potential outcomes of sporting events affects odds set by bookmakers, using a generalized version of the model in Shin (1991). The model has been widely cited as an explanation for the pattern of favorite-longshot bias observed in fixed-odds betting markets. We show that disagreement among those bettors without inside information causes favorite-longshot bias. The presence of insiders reduces odds but does not necessarily exacerbate favorite-longshot bias. For realistically calibrated beliefs, the fraction of insiders has a minimal effect on the ratio of favorite to longshot odds and the betting market collapses if this fraction rises above low levels.

Keywords: Sports Betting, Inside Information

JEL Classification: G14, L83, Z20

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1. Introduction

This paper examines how betting odds set by bookmakers are affected by the presence of insiders with superior information, focusing in particular on whether insiders have much influence on the relative odds of favorites and longshots. We present a generalization of Shin's (1991) model of fixed-odds bookmaking using a less restrictive formulation of the beliefs of those who do not have inside information and use the model to show that the fraction of insiders has either no impact or almost no impact on these relative odds. We also discuss the separate model of this situation presented by Shin (1992) and argue that it does not provide a convincing explanation for insiders as an influence on the relative odds of favorites and longshots.

This topic is of interest for two reasons. First, sports betting via bookmakers is growing rapidly around the world, making it more important to understand these markets. While most research on sports betting has focused on pari-mutuel betting (which pools bets and then allocates most of the pool to those who backed the winner) the rapid expansion of recent years, facilitated by mobile internet technology and by legal changes such as the 2018 US Supreme Court ruling that overturned federal restrictions on sports betting, has been driven by fixed-odds betting with bookmakers.[1]

The literature on fixed-odds betting has documented a consistent pattern of average payouts on favorites being larger than for longshots. This finding has been reported for UK horse racing (Gabriel and Mardsen, 1990, Snowberg and Wolfers, 2010), US football and basketball (Berkowitz, Depken and Gandarc, 2017, Moscowitz and Vasudevan, 2022), European soccer (Buhagiar, Cortis and Newell, 2018, ngelini and de ngelis, 2019) and tennis (Forrest and McHale, 2007, Lahvička, 2014). Shin's (1991, 1992) models have often cited as illustrating how the presence of insiders generates this favorite-longshot bias. For example, these models were cited in literature reviews by Ottaviani and and Sørensen (2008) and Snowberg and Wolfers (2008) as the principle examples of how favorite-longshot bias can emerge from fixed-odds betting markets. Our results shed light on the strength of the theoretical case for this idea.

Second, the topic also relates to the behavior of competitors in sporting events. While it is possible that significant amounts of money are placed with bookmakers by people who use publicly available information to come up with better probability assessments than bookmakers, the other explanation that has often been cited relates to corruption, so that some bettors have bribed officials or players to influence the result. Corruption of this kind has been sometimes been explicitly uncovered, from the infamous "Black Sox" scandal of the 1919 World Series to modern examples in Italian soccer (Boeri and Severgnini, 2011). However, by its nature, it is hard to know how prevalent this kind of activity is. If the pattern of favorite-longshot bias that has been documented across a wide range of major sports is driven by the presence of insiders, this would suggest corruption may be common even for

[1] Fixed-odds betting features bookmakers making offers such as "You will take back $3 if your pick wins and lose your $1 bet otherwise" and payouts on successful bets don't depend on how much money other bettors placed.

high-stakes sporting events.

Our generalization of Shin's 1991 model can be explained as follows. In relation to a two-team event, the model assumed that non-insiders held beliefs $\tilde{p}$ about the probability of one of the teams winning that were randomly drawn from a uniform distribution on $[0, 1]$. This means their beliefs were unrelated to the true value of the probability. We argue that this assumption is unrealistic. Even if most people know little about sports, those willing to bet on sports events will usually have at least some relevant information about the relevant probabilities. Beliefs being unrelated to reality also generates counter-factually high average loss rates for bettors: Bettors with no useful information turn out to lose large fractions of their money. We present a generalized model in which beliefs are uniformly distributed on $[A, B]$, paying particular attention to the case where $\tilde{p}$ is drawn from a uniform distribution over $[p - \sigma, p + \sigma]$ where p is the true probability. This "wisdom of crowds" approach, in which the public are on average correct in their assessment of the relevant probability, has been used in the literature on betting since Ali (1977).

We show that the presence of insiders reduces odds but does not necessarily produce favorite-longshot bias. When $A + B = 1$ (which includes the model in Shin, 1991) the proportional reduction in odds on the favorite is the same as for the longshot, so the ratio of these odds does not depend on the fraction of insiders. For more realistically calibrated beliefs, an increase in the fraction of insiders reduces longshot odds proportionately more than favorite odds, but the effect is minimal. We also show that when beliefs are realistically calibrated, the market collapses once the fraction of insiders rises above a relatively low percentage because optimal profits for an operating bookmaker are negative, meaning it will choose not to enter the market. This provides an additional reason for skepticism about the presence of insiders as an important explanation for favorite-longshot bias.

We also briefly discuss Shin's (1992) alternative model of insiders in sports betting. This model has a different structure to the 1991 model and it does imply that the fraction of insiders induces favorite-longshot bias. However, we argue that the key features of this model—most notably that the demand for bets is not sensitive to the odds—provide a poor description of the real-world sports betting business and thus its explanation for this bias is not credible.

The paper is structured as follows. Section 2 illustrates the evidence on favorite-longshot bias in fixed odds betting using data on soccer and tennis. Section 3 presents the generalized version of the Shin (1991) model and Section 4 provides graphical illustrations of the results. Section 5 discusses Shin's (1992) model and Section 6 concludes.

2. Evidence on Favorite-Longshot Bias

It is useful to start with empirical examples that illustrate the scale and pattern of the favorite-longshot bias in fixed odds betting markets. We use two datasets made publicly available by gambling expert, Joseph Buchdahl. From www.football-data.co.uk, we have betting odds and outcomes on 84,230 European professional soccer matches, spanning the 2011/12 to 2021/22 seasons for 22 European soccer leagues across 11 different nations. The leagues covered are listed in an appendix. From www.tennis-data.co.uk, we have odds and outcomes for all 58,112 professional men's and women's matches played across the world on the TP and WT tours between 2011 and 2022. Our measure of betting odds is the average closing odds across the wide range of online bookmakers surveyed by Buchdahl.

Favorite-longshot bias can be illustrated by sorting the bets available by an estimate of their perceived likelihood of success. We do this sorting as follows. Let N be the number of possible outcomes in an event. This is two in the case of the tennis data and three for the soccer data because soccer matches can end in draws (ties). Let p_i be the probability that a bet on outcome i will be successful and O_i be the decimal odds for this bet, meaning the total payout from betting $1 on outcome i when this outcome occurs, inclusive of the original bet. We assume these satisfy

$$p_i O_i = \mu \qquad i = 1, ..., N \tag{1}$$

where $\mu < 1$. This assumption of a constant expected return across all bets on the same event fits Thaler and Ziemba's (1988) definition of an efficient betting market. These conditions combine with the condition that the probabilities sum to one to provide $N + 1$ linear equations which solve to give a unique set of $N + 1$ unknown values, namely the N probabilities and an expected payout μ. Specifically, μ is given by

$$\mu = \frac{1}{\sum_{i=1}^{N} \frac{1}{O_i}} \tag{2}$$

and, given μ, the so-called normalized probabilities can then be calculated directly from equation 1. The expected payout is determined by the sum of the inverses of the odds. The extent to which this sum exceeds one is known in bookmaking circles as the overround.

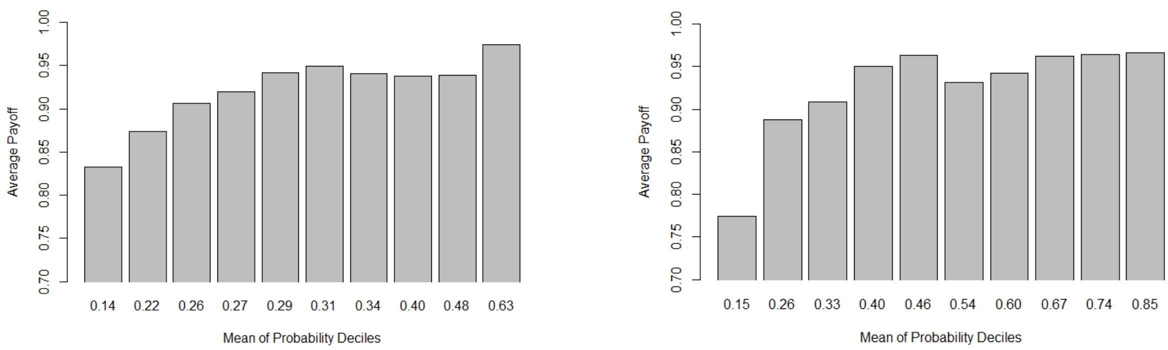
Figure 1 displays two charts that divide all bets in our samples into deciles by their predicted probability of success. For each decile, the charts display the average payout on these bets per dollar staked. For the soccer dataset, bets in the lowest decile for estimated probability of success have an average payout on a $1 bet of only $0.83 (meaning an average loss of 17%) while bets in highest decile have only a 2% average loss rate. For the tennis dataset, the pattern is even more extreme, with bets in the bottom decile losing 23% on average while bets in the top decile lose only 3%. This evidence confirms the existing findings using smaller datasets of Buhagiar, Cortis and Newell (2018) and ngelini and de ngelis (2019) for soccer and Forrest and McHale (2007) and Lahvička (2014) for tennis. The fact that bets in different deciles have different average returns suggests the identifying assumption of equal expected returns in equation 1 is incorrect. However, theoretical models that generate favorite-longshot bias, such as those we will present here, still predict that probability estimates calculated in this manner will rise monotonically with the true probability, so the patterns in the bar charts would look the same if we applied a method to recover probabilities that accounted for differing average payouts across bets.

In addition to noting the pattern of favorite-longshot bias observed in the data, it will also be useful to note that the average overrounds are 6.9% for the soccer data and 5.8% for the tennis data. We will return to these figures later when discussing the calibration of the theoretical models.

Figure 1: verage payout rates for bets by deciles of estimated values of the probability the bet will win

(a) Soccer

(b) Tennis



3. Generalized Model with Insiders and Disagreement

Here we present a generalization of Shin's (1991) model, featuring a monopoly bookmaker, insiders with superior knowledge and non-insiders who disagree about the probabilities of different outcomes. We start by illustrating the general principles underlying odds-setting by a monopoly bookmaker.

3.1. Economic principles of monopoly bookmaker odds

Consider the following general model of a monopolistic bookmaker, similar to the model in Montone (2021). The bookmaker offers a bet that will pay decimal odds O if an event occurs. The event will occur with a probability p, which is known to the bookmaker, of the bet being successful. There is a demand function for this bet $D(O)$ such that $D'(O) > 0$ and $D''(O) < 0$. There is an administrative cost to the bookmaker for each bet equal to a fraction μ of the amount placed. The bookmaker's expected profit on the bet is

$$E(\quad) = (1 \quad \mu) D(O) \quad pO(D(O)) \tag{3}$$

The condition for maximizing expect profit is

$$\frac{dE(\quad)}{dO} = (1 \quad \mu \quad pO) D'(O) \quad pD(O) = 0 \tag{4}$$

Defining the elasticity of demand for the bet as

$$\epsilon = \frac{OD'(O)}{D(O)} \tag{5}$$

optimal odds can be written as

$$O = \frac{\epsilon}{\epsilon + 1} \frac{1 \quad \mu}{p} \tag{6}$$

Expected profits are zero when odds are $\frac{1}{p}$. The bookmaker thus chooses to set odds as a "markdown" on the zero-profit odds and bets with higher elasticities of demand will have lower markdowns, with odds tending towards the zero expected profit odds as the elasticity of demand increases.

3.2. disagreement-based model

We consider an event with two teams where one of them will win. One of the teams is the favorite and has a probability $p \geq 0.5$ of winning; the other team we will term the longshot. The bookmaker knows the value of p and sets decimal odds of O_F on the favorite and O_L on the longshot to maximize their expected profits. We retain the assumption, not in Shin's (1991) model, of an administrative cost of μ per bet.

The total amount of wealth available to be placed as bets is normalized to 1.   fraction z of the wealth is held by bettors who are insiders who know the outcome of the event and always win their bets. The rest is held by a continuum of non-insiders of size 1 who can chose to bet or not. When they do bet, they bet one unit. These bettors disagree with each other about what the probability p is, having subjective beliefs $\tilde{p}$ that are drawn independently from a uniform distribution on $[A, B]$.[2] Shin (1991) assumed $A = 0$ and $B = 1$. Non-insiders are risk-neutral and only bet if they perceive the expected return to be at least zero.

The formulation that insiders know the result is best understood as an extreme interpretation of the model. The model could be generalized so that insiders do not know the outcome of the event for sure but just have a more accurate assessment than non-insiders. For example, if we assumed that a fraction Mz of wealth was held by bettors who are randomly correct about the outcome one-Mth of the time and the rest of the time had the same randomly-distributed beliefs as non-insiders, we would obtain the same results that we derive here. So we will proceed with the assumption of insiders who know the outcome, understanding that z is best seen as a proxy for the fraction of wealth placed on winning bets by bettors with superior knowledge.[3]

The condition for non-insiders to be willing to bet on the favorite is

$$\tilde{p}O_F \geq 1 \Longrightarrow \tilde{p} \geq \frac{1}{O_F} \tag{7}$$

This generates a demand function for bets on the favorite from non-insiders of the form

$$D\left(O_F\right) = 1 \quad \frac{1}{B \quad A}\left[\frac{1}{O_F} \quad A\right] \quad \text{for} \quad \frac{1}{B} \leq O_F \leq \frac{1}{A} \tag{8}$$

For odds above $\frac{1}{A}$ all non-insiders take the bet so their demand equals 1 and for odds below $\frac{1}{B}$, none of them take the bet.

Bets are placed after the bookmaker has set the odds and the bookmaker is aware that a fraction z of wealth will be placed on the winner by insiders. Given these assumptions, the bookmaker's expected profit from taking bets on the favorite is

$$E\left(\quad _F\right) = pz\left(1 \quad \mu \quad O_F\right) + \left(1 \quad z\right)\left[\left(1 \quad \mu\right) D\left(O_F\right) \quad pO_F\left(D\left(O_F\right)\right)\right] \tag{9}$$

The bookmaker sets odds to maximize the expected profit on each type of bet. Combining the demand for bets on the favorite with the expected profit equation (and assuming odds that generate

[2]Uniform distributions for beliefs are convenient for generating analytical formulas but solving the model numerically with Normally-distributed beliefs produces essentially the same results as reported here.

[3]This more general interpretation of   was first proposed by Fingleton and Waldron (1996)

non-insider demands between zero and one) the first-order condition for expected profit maximization is

$$\frac{\partial E(\;)}{\partial O_F} = \quad zp + (1 \quad z)(1 \quad \mu)\left(\frac{1}{B \quad A}\right)\frac{1}{O_F^2} \quad \frac{(1 \quad z)\,p}{B \quad A} = 0 \tag{10}$$

which solves to give

$$O_F = \sqrt{\frac{1 \quad \mu}{p\left(\frac{z\;B \quad A)}{1 \quad z} + B\right)}} \tag{11}$$

The bookmaker's problem for setting odds on the longshot is the same apart from the true probability being $1 \quad p$ instead of p and beliefs are drawn from a uniform distribution on $[1 \quad B, 1 \quad A]$ instead of $[A, B]$. The odds on the longshot are thus

$$O_L = \sqrt{\frac{1 \quad \mu}{(1 \quad p)\left(\frac{z\;B \quad A)}{1 \quad z} + 1 \quad A\right)}} \tag{12}$$

So the ratio of odds on the favorite team to odds on the longshot is

$$\frac{O_F}{O_L} = \sqrt{\frac{(1 \quad p)\left(\frac{z\;B \quad A)}{1 \quad z} + 1 \quad A\right)}{p\left(\frac{z\;B \quad A)}{1 \quad z} + B\right)}} \tag{13}$$

We will summarize five properties of these odds.

1. Insiders lower odds: Higher values of z lower the odds for both teams. This is because the presence of insiders, who will bet no matter what the odds are, reduces the elasticity of demand for bets, leading the bookmaker to charge higher markdowns.

2. Insiders don't necessarily affect the ratio of odds: Consider the case $A+B = 1$, so the distribution of beliefs for non-insiders is centered on 0.5. In this case, the ratio of odds reduces to

$$\frac{O_F}{O_L} = \sqrt{\frac{(1 \quad p)}{p}} \tag{14}$$

Note that z does not feature in this formula, so the amount of bets by insiders has no effect on the ratio of the odds of the two teams. Equation 14 is the formula presented by Shin (1991) for favorite-longshot bias. This result was obtained because Shin's assumption of uniform beliefs on $[0, 1]$ is a special case of $A + B = 1$. Shin's paper has often been interpreted as an example of favorite-longshot bias being a response of bookmakers to the presence of insiders but, in fact, the specific case it examined implied the fraction of wealth won by insiders had no impact on the relative odds.

3. Disagreement generates favorite-longshot bias: Equation 14 shows the model can generate favorite-longshot bias independent of how many insiders there are because the ratio of the odds is determined by the square root of the ratio of probabilities rather its level. To give an example, suppose $p = 0.7$. If the expected return on both bets was the same, then the ratio of favorite odds to longshot odds would be determined by the relative probability as $0.43 = \frac{0.3}{0.7}$. Equation 14 implies the bookmaker sets a ratio of favorite odds to longshot odds of 0.65 when $A + B = 1$, meaning the relative odds for the favorite are better and thus the return for betting on the favorite is higher than for betting on the longshot.

What generates this favorite-longshot bias since it is not due to insiders? The answer is that disagreement among non-insiders generates varying elasticities of demand across different bets, a result previously presented for a model without insiders by Hegarty and Whelan (2023).

This point can be illustrated beyond the specific case of $A + B = 1$. For example, suppose non-insider beliefs are uniform on $[0.6, 0.8]$ and the true $p = 0.7$. The zero-profit odds with no insiders and $\mu = 0$ would be $O_F = \frac{1}{0.7} = 1.43$ and $O_L = \frac{1}{0.3} = 3.33$. t these odds, half the potential bettors (those with $\tilde{p} \geq 0.7$) would take the bet on the favorite and the other half would take the bet on the longshot. Now suppose the odds on both teams were cut by 10%. t the new favorite odds of $O_F = 1.29$ only those with $\tilde{p} > 0.777 = \frac{1}{1.29}$ will take the bet, meaning only 11% of potential bettors now bet on the favorite. In contrast, at the new longshot odds of $O_L = 3$, those with $1 - \tilde{p} > \frac{1}{3}$, meaning $\tilde{p} < 0.667$ will take the bet, meaning 33% of potential bettors will still take the longshot bet. This illustrates the weaker sensitivity to odds of demand for longshot bets.

4. For realistic beliefs, bias without insiders will be smaller than implied by Shin's formula: Shin's model assumed that bettors without inside information had no useful knowledge about the relative merits of the teams. However, while it may be true that there are many people who don't have an informed opinion about sporting events, the relevant population for this model is people who are willing to make bets on the game and these are more likely to have opinions that are somewhat informed by reality. t a minumum, it seems likely that the non-insider public is sufficiently well-informed so that its mean belief is greater than 0.5 when $p > 0.5$. In other words, on average, the public understands that one team is more likely to win than the other. This means $A + B > 1$ and thus $B > 1 - A$.

To see the impact of more realistic beliefs, let us set $z = 0$, so the odds ratio is

$$\frac{O_F}{O_L} = \sqrt{\frac{(1 - p)(1 - A)}{pB}} \tag{15}$$

We can see that the more realistic beliefs imply a smaller value for $\frac{O}{O_L}$ than Shin's formula and thus a weaker pattern of favorite-longshot bias.

useful benchmark for realistic beliefs is to assume, as in li (1977), that the the public are, on average, correct about the probability of each team winning so beliefs are uniform on $[p - \sigma, p + \sigma]$. In this case, the ratio of odds with $z = 0$ becomes

$$\frac{O_F}{O_L} = \sqrt{\frac{(1 - p)(1 - p + \sigma)}{p(p + \sigma)}} \tag{16}$$

The extent of favorite-longshot bias depends positively on σ, the parameter that determines the amount of disagreement about the probability amongst insiders. Higher levels of disagreement have more impact in reducing the elasticity of demand for the longshot than in reducing it for the favorite. Note that this formula suggests that if $\sigma = 0$, the odds ratio would be the same as for fairly-set odds i.e. odds would be inversely proportional to probabilities. However, in this case, the market would collapse as long as $\mu > 0$ because none of the now-certain bettors would believe they could make a profit given the bookmaker's need to cover costs.

5. Insiders will usually exacerbate the favorite-longshot bias: For most realistic specifications of beliefs, a larger fraction of insiders means a larger favorite-longshot bias. We can show this by differentiating the odds ratio formula in equation 13 with respect to z

$$\frac{\partial\left(\frac{O}{O_L}\right)}{\partial z} = \frac{1}{2}\sqrt{\frac{O_L}{O_F}}\frac{p(1 - p)(B - A)(A + B - 1)}{p^2\left[\frac{z}{1-z}(B - A) + B\right]^2}\frac{1}{(1 - z)^2} \tag{17}$$

This is positive as long as $A + B > 1$, so the public on average understands that the stronger team is the favorite. However, as we will see, the the impact of z on relative odds is small.

4. Illustrating the Results

Here we use calibrated examples to illustrate the results and to describe how the bookmaker's awareness of the presence of insiders can cause the market to collapse.

4.1. Odds, overrounds and payout rates

Figure 2 shows the odds generated by the Shin (1991) model's assumption that non-insider beliefs are uniformly distributed over $[0, 1]$ for a range of estimates of p with the cost parameter set at $\mu = 0.02$. The top and middle panels show odds for favorites and longshots as a function of the probability of the favorite winning for various different values of z ranging from zero to 0.1. For comparison, the figure also shows the no-insider zero-profit odds defined by

$$O_F = \frac{1 \quad \mu}{p} \qquad O_L = \frac{1 \quad \mu}{1 \quad p} \tag{18}$$

The figure shows that the odds set by the monopolist bookmaker are a lot lower than zero-profit odds and that the odds for both favorites and longshots depend negatively on z. However, the size of this impact is pretty small. The bottom panel shows the ratio of favorite odds to longshot odds as a function of z for various values of p. The lines are flat because for this specification of beliefs, z has no impact on the extent of favorite-longshot bias.

Figure 3 illustrates the odds generated by the more realistic assumption that non-insider beliefs are uniformly distributed over $[p \quad \sigma, p + \sigma]$ where in this case $\sigma = 0.06$. Odds are significantly higher because the public's more accurate beliefs make demand more sensitive to the odds. gain we can see that z has a relatively small impact on odds. The lines in the bottom panel slope up very slightly indicating higher values of z have an impact in raising favorite odds relative to longshot odds but the effect is tiny. For example, for $p = 0.6$, $\frac{O}{O_L} = 0.6816$ when $z = 0$ and equals 0.6846 when $z = 0.1$.

Figures 4 and 5 illustrate why the model with uniform beliefs on $[0, 1]$ is far less realistic than the "wisdom of crowds" assumption that the public is on average correct. Figure 4 shows the overrounds generated by the model, calculated as

$$R = \frac{1}{O_F} + \frac{1}{O_L} \quad 1 \tag{19}$$

The upper panel shows that for uniform beliefs on $[0, 1]$, the overrounds range from 30% to 50%, which is far higher than observed in actual betting markets. The odds are so poor because many non-insiders have wildly inaccurate beliefs and are thus willing to accept really poor odds. When beliefs are uniformly distributed over $[p \quad \sigma, p + \sigma]$, the overround depends on the value of σ. s disagreement rises, we get more bettors with inaccurate beliefs and this allows the bookmaker to set odds that deviate more from the zero-profit levels. We have set $\sigma = 0.06$ so that the model's overround without insiders roughly matches the data described above.

Figure 5 shows the average payout rates for non-insiders, meaning the fraction of a \$1 bet that will be returned on average (pO_F for favorites and $(1 - p)\,O_L$ for longshots). The upper panel shows payout rates are low for all bets when beliefs are uniform on $[0, 1]$ but particularly low for longshot bets. The average loss rate for bets with a 15% chance of winning is a catastrophic 62% when there are no insiders and even higher when z is positive while loss rates for bets with a 90% chance of winning are about 6%. In contrast, for non-insider beliefs that are uniform on $[p - 0.06, p + 0.06]$ and $\mu = 0.02$, the minimum loss rate for bets with a 15% chance of winning is 16% and for bets with a 90% chance of winning is 4%. These lower loss rates are more consistent with the evidence presented for soccer and tennis betting in Section 2 and also with the evidence of a nonlinear pattern that sees loss rates accelerating for bets with lower probabilities of winning.

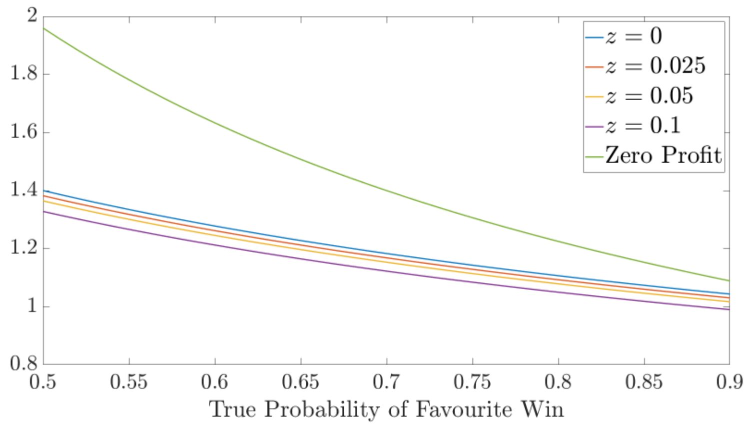
4.2. Profit rates and market collapses

Figure 6 shows the expected profit rate for bookmakers on matches (i.e. profits divided by the total dollar amount of bets placed) as a function of the probability of the favorite winning and for various values of z. The upper panel shows profit rates are high when beliefs are uniform on $[0, 1]$, particularly when there is a strong favorite. higher fraction of betting from insiders reduces profit rates but even with 10 percent of bets being placed by insiders, average profit rates on games are always high. In contrast, the lower panel of Figure 6 shows much lower levels of profitability when beliefs are more realistically calibrated uniform on $[p - 0.06, p + 0.06]$.

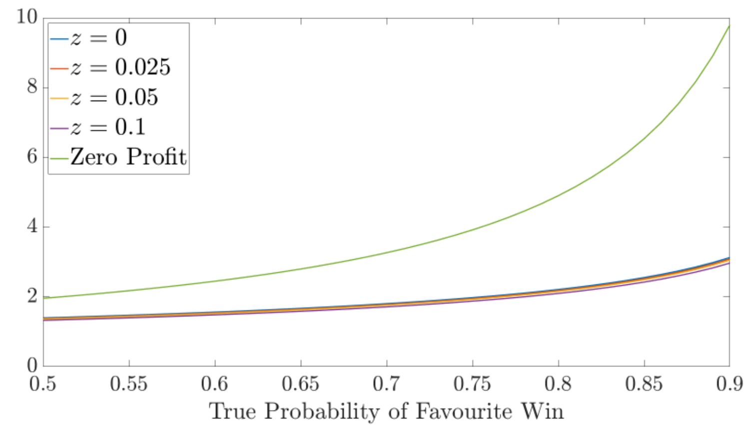
Note that our formula for the profit-maximizing odds did not guarantee that the optimal level of profits for an operating bookmaker would be positive. In fact, it turns out that these "optimal" odds will often generate negative profits, meaning the actual optimal strategy for the bookmaker is to not take bets at all. If 2.5% of bets are from insiders, only games where the favorite has a probability of winning over 0.67 are profitable. If 5% of bets are from insiders, only games where the favorite has a probability of winning over 0.85 are profitable and for higher amounts of insiders, all games are loss making. Effectively, the betting market collapses when the fraction of bets placed by insiders rises above a small fraction because losses inflicted on non-insiders with realistic beliefs can't make up for the profits earned by insiders.

Figure 2: Monopoly odds and zero profit odds (with no insiders) for various values of p and ratio to favorite odds to longshots for various values of z. Non insiders have beliefs about p that are uniform on $[0, 1]$ and $\mu = 0.02$

(a) Favorite odds for monopolist and competitive odds



(b) Longshot odds for monopolist and competitive odds



(c) Ratio of favorite odds to longshot odds

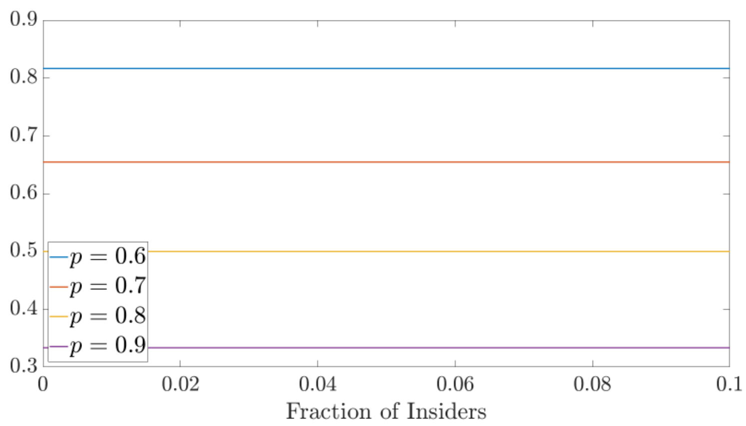
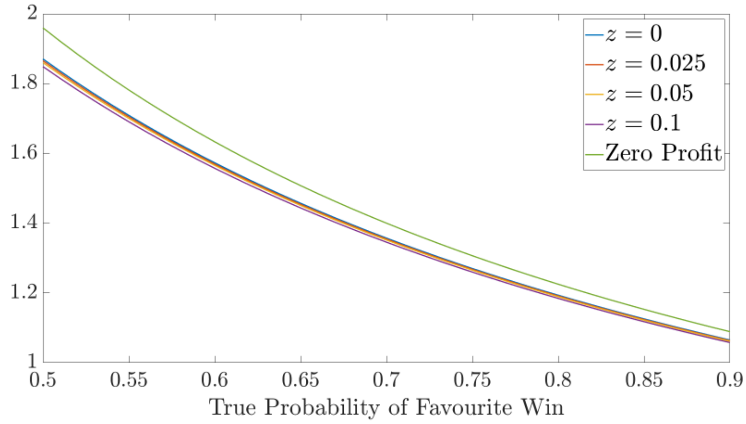
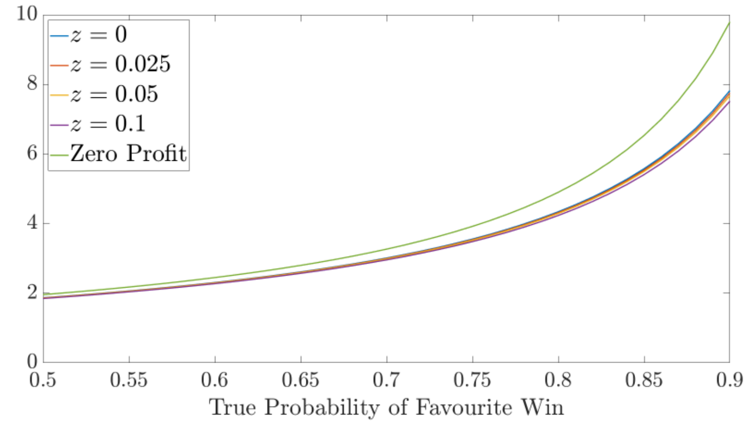


Figure 3: Monopoly odds and zero profit odds (with no insiders) for various values of p and ratio to favorite odds to longshots for various values of z. Non insiders have beliefs about p that are uniform on $[p \quad 0.06, p + 0.06]$ and $\mu = 0.02$

(a) Favorite odds for monopolist and competitive odds



(b) Longshot odds for monopolist and competitive odds



(c) Ratio of favorite odds to longshot odds

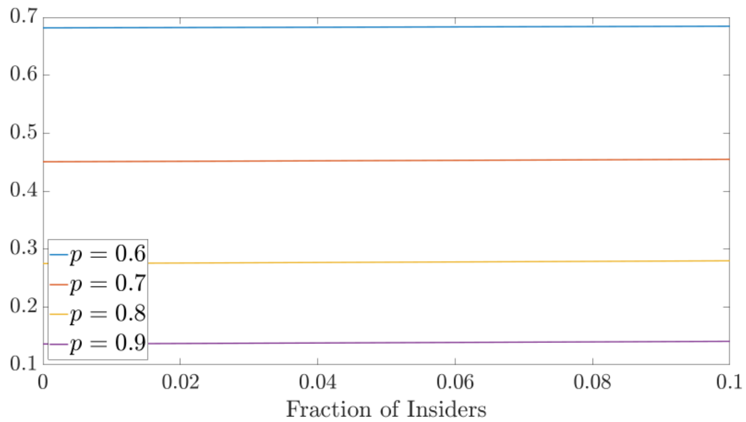
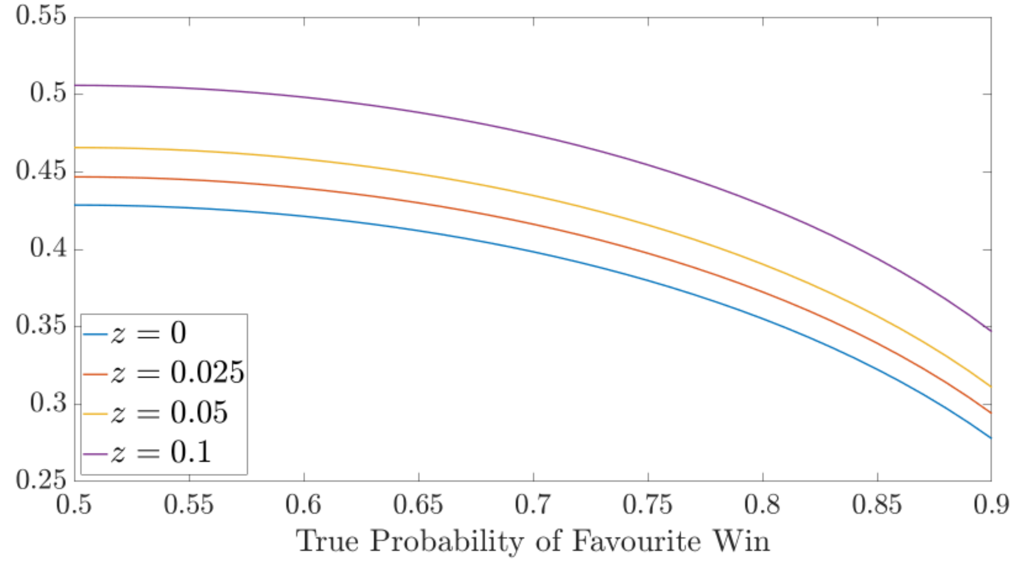


Figure 4: Overrounds for various values of p and z with $\mu = 0.02$.

(a) Beliefs are uniform on $(0, 1)$



(b) Beliefs are uniform on $(p - 0.05, p + 0.05)$

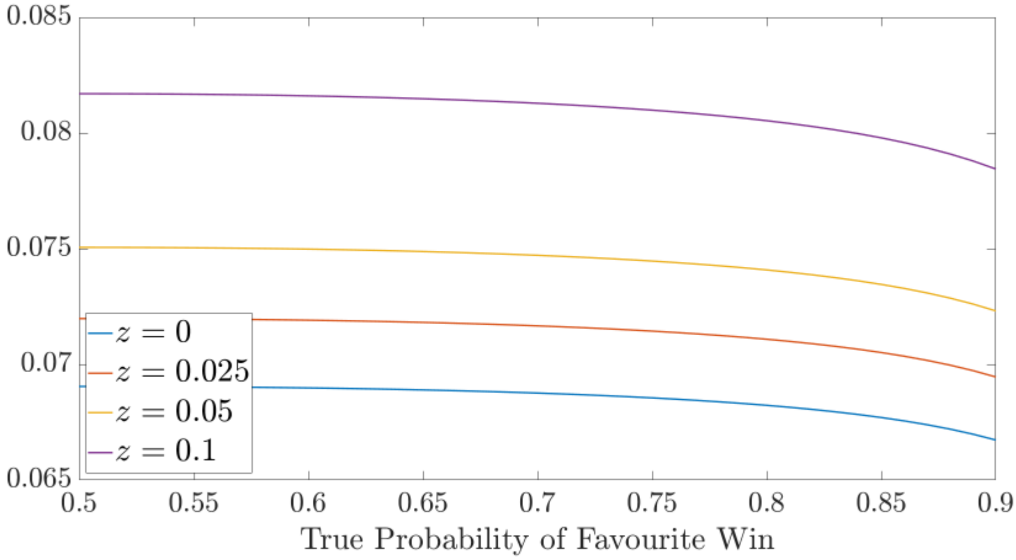
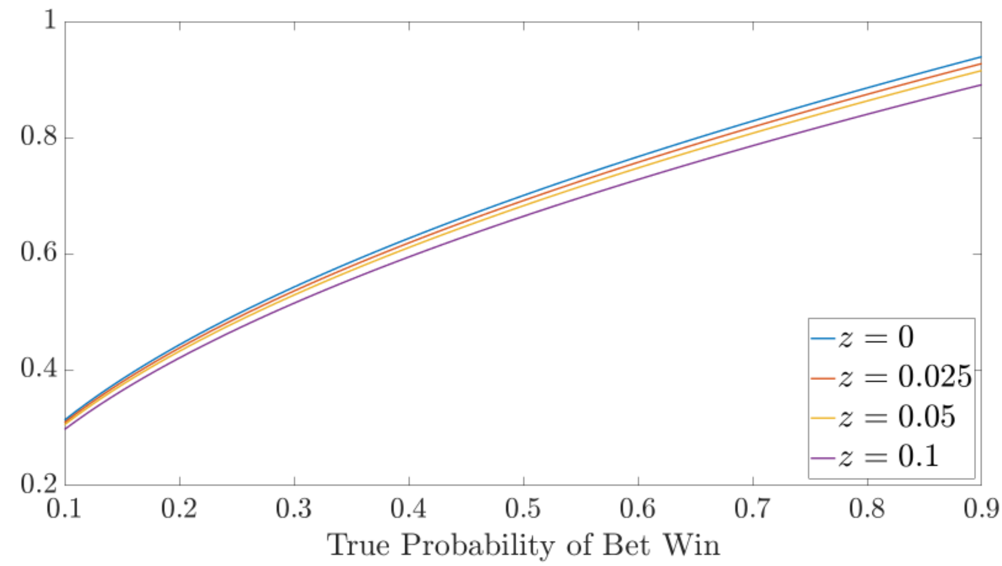


Figure 5: verage payout rates for non-insiders for various values of z and the probability of bet success with $\mu = 0.02$.

(a) Beliefs are uniform on $(0, 1)$



(b) Beliefs are uniform on $(p \quad 0.06, p + 0.06)$

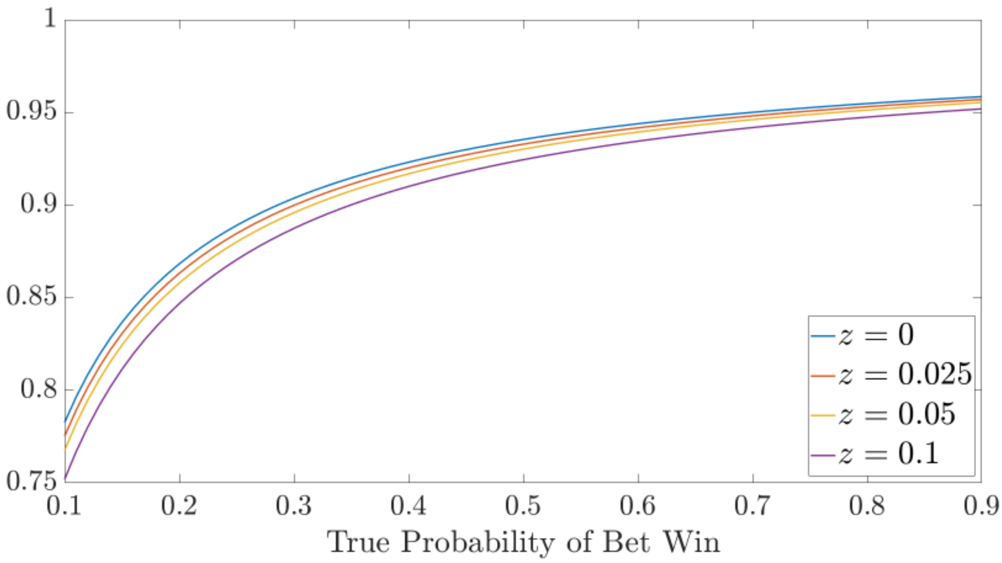
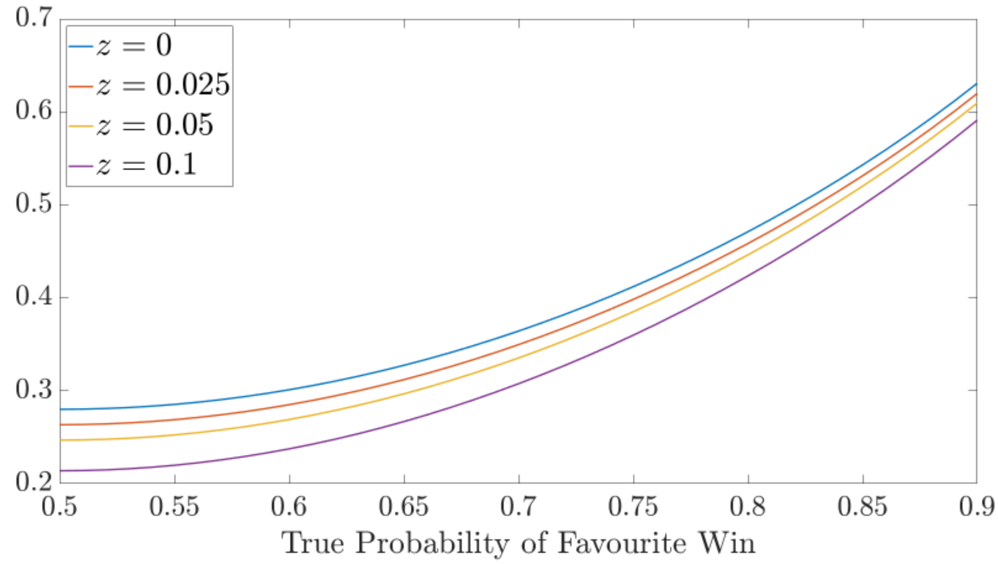
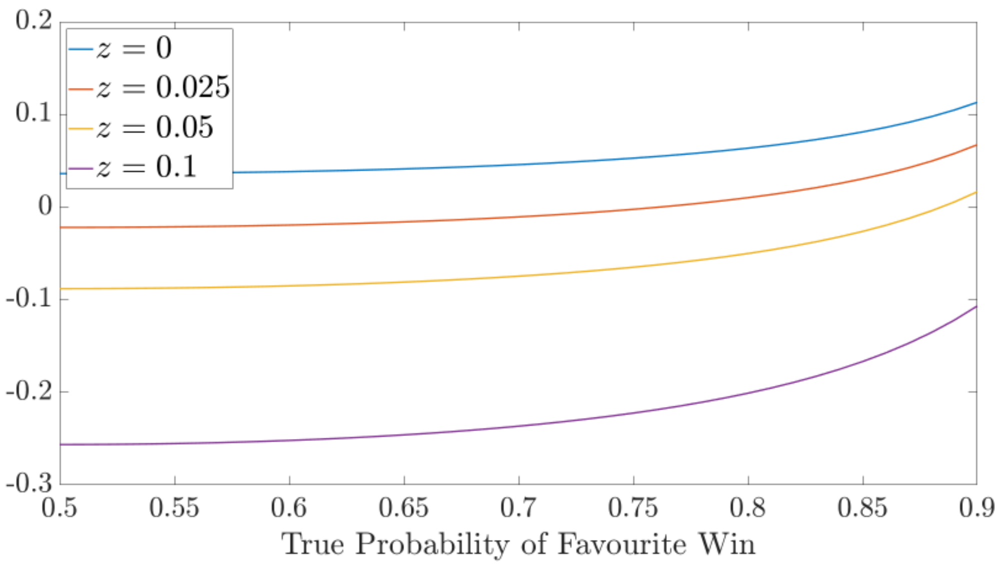


Figure 6: Monopoly average profit rate on matches for various values of p and z with $\mu = 0.02$.

(a) Beliefs are uniform on $(0, 1)$



(b) Beliefs are uniform on $(p - 0.06, p + 0.06)$



5. The Shin (1992) Model

Shin (1992) provided an alternative model of how insiders can influence odds set by bookmakers. Here, we briefly describe this model and discuss its applicability to modern online sports betting. The 1992 model assumes there are N competitors in a sporting event in which only one can win. There is a continuum of bettors of size 1, each making equally sized bets, also normalized to 1. gain, a fraction z of bettors are insiders who know which competitor is going to win. The remaining bettors are each certain that one of the competitors will win, with the fraction believing competitor i will win equal to the probability p_i that it will win. This means the potential bettors do not consider the odds other than requiring that they do not lose money when their bet wins.

The bookmaker is a monopolist. No costs are incorporated into the model other than paying out on bets, so the bookmaker's expected profit is

$$E(\ ) = 1 \quad \sum_{i=1}^{N} zp_iO_i + (1 \quad z)\,p_i^2O_i] \tag{20}$$

where O_i is the odds on the ith competitor. With all bettors willing to bet for any odds better than 1, the unconstrained profit maximizing odds would be $O_i = 1$ for all i, implying an overround of $N \quad 1$. However, there is an exogenously-set limit on the overround, so that

$$\sum_{i=1}^{N}\frac{1}{O_i} \leq \tag{21}$$

This constraint could be interpreted in different ways. Shin (1992) describes a process by which potential bookmakers bid for a monopoly license and the body awarding the license (presumably some arm of government) gives it to the the bookmaker that submits the lowest value of , thus providing the best value for bettors. n alternative interpretation is that the bookmaker is not a monopolist. Instead, competitive pressures set the value of and, once this value is set, each bookmaker obtains the same fraction of bets on each competitor.

Given this constraint, the bookmaker sets odds by optimizing the Lagrangian

$$L = 1 \quad \sum_{i=1}^{N} zp_iO_i + (1 \quad z)\,p_i^2O_i] + \lambda\left(\quad \sum_{i=1}^{n}\frac{1}{O_i}\right) \tag{22}$$

The first-order conditions are

$$\frac{\partial L}{\partial O_i} = \quad zp_i \quad (1 \quad z)\,p_i^2 + \frac{\lambda}{O_i^2} = 0 \tag{23}$$

and that the constraint in equation 21 is binding. Optimal odds are

$$O_i = \sqrt{\frac{\lambda}{zp_i + (1 - z)\, p_i^2}} \tag{24}$$

This implies the ratio of the odds on competitor i to competitor j is

$$\frac{O_i}{O_j} = \sqrt{\frac{zp_j + (1 - z)\, p_j^2}{zp_i + (1 - z)\, p_i^2}} = \frac{p_j}{p_i}\sqrt{\frac{1 - z + z/p_j}{1 - z + z/p_i}} \tag{25}$$

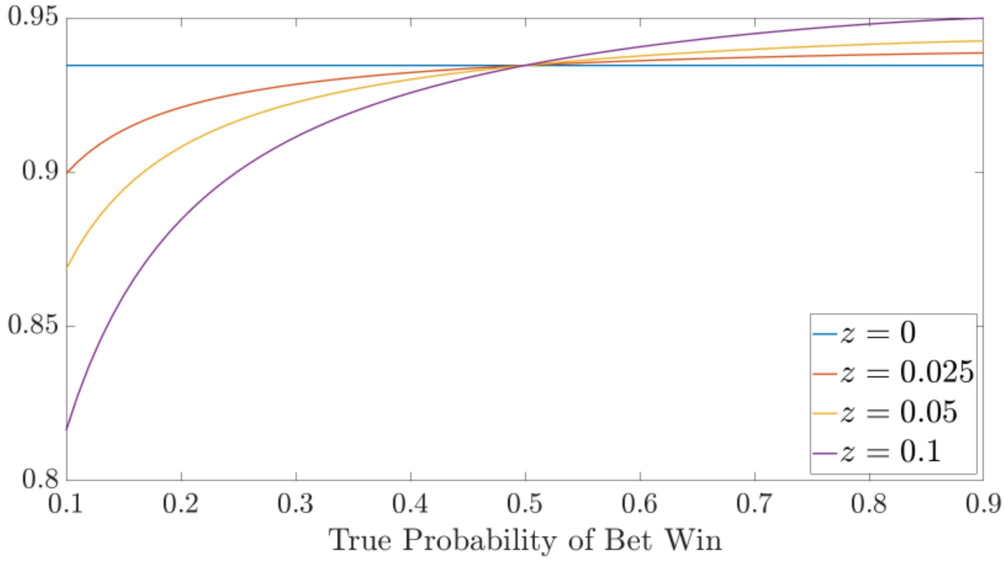
If $p_i > p_j$, then $z/p_i < z/p_j$ and $1 - z + z/p_i < 1 - z + z/p_j$ meaning $\frac{O_i}{O_j} > \frac{p_j}{p_i}$ which means there is a favorite-longshot bias.

To illustrate this bias, consider the case where the overround constraint is set at $\lambda = 1.07$, which is roughly consistent with the evidence on overrounds shown above. Figure 7 shows the payout rates for bets generated by this model as a function of their probability of success for several values of z. As z increases, the pattern of favorite-longshot bias strengthens and the shape of the payout rate function becomes more like the nonlinear pattern documented in Section 2. That said, with this calibration, one needs a relatively large (and possibly incredible) value for the fraction of insider money, at least 10 percent, to match the size of losses on extreme longshots seen in the data.

So the model matches some aspects of the real-world data. But does it provide an adequate description of the modern online sports betting industry? There are several ways in which it clearly does not. Most importantly, the lack of any influence of the odds on the behavior of bettors (a key feature of the 1991 model) means the model lacks the crucial feature that bookmakers need to consider when setting odds. That potential bettors are highly sensitive to odds can be seen in the popularity of special odds-boosting offers that bookmakers make in order to attract more business. Natural experiments such as the reform of betting tax in the UK in 2001 have also shown that demand for betting is closely linked to bettors' perceptions of potential financial returns. As described by Paton, Siegel and Vaughan Williams (2002, 2004), the UK's decision to abolish a "general betting duty" levied on all bettors as they placed their bets and replace it with a profit tax for bookmakers reduced the net loss rate from betting and led to a doubling in off-site betting activity over the next year.

Other devices in the model such as the externally-imposed overround condition, and the coincidence of the fraction of people who believe competitor i is going to win exactly matching the probability that this competitor will win, are also questionable. Shin (1992) notes that if $z = 0$, this latter assumption produces the same odds as a pari-mutuel betting market but it is unclear why this is relevant for this model of fixed-odds betting. The assumption seems a less adequate formulation of the "wisdom of crowds" idea than the approach we used for the previous model in which people disagree about probabilities but are, on average, correct. Overall, this model seems unlikely to provide a useful description of how modern bookmakers set odds.

Figure 7: Payout rate for Shin (1992) model for various values of p and z with $\ = 1.07$.



6. Conclusions

The idea that the favorite-longshot bias observed in fixed-odds betting markets results from bookmakers reacting to the presence of insiders with superior knowledge been widely referenced since Shin's sequence of papers in the early 1990s. We have shown that this idea has limited theoretical support. In models where bettors disagree about the underlying probabilities and the demand for bets is sensitive to the odds offered, the presence of insiders has a very limited impact on the balance of odds between favorites and longshots and in some cases has no impact.

We have also provided another reason to be skeptical about insiders as a cause of favorite-longshot bias. Bookmakers being aware of insiders leads them to reduce odds. Once non-insider bettors have realistic beliefs and their demand for bets is sensitive to the odds, even a relatively small fraction of insiders is sufficient for a betting market to collapse. The low odds that bookmakers have to offer to allow them to break even become unacceptable to non-insiders.

n exception to these theoretical results is the model presented in Shin (1992) but the betting market in this model looks almost nothing like modern real-world betting markets. In particular, its assumption that the actions of the bettors do not depend on the odds set by bookmakers omits probably the most important feature of real-world betting markets. We conclude that while there are many possible explanations for favorite-longshot bias in fixed-odds betting markets—and the model presented here suggests disagreement among bettors combined with market power for bookmakers provides one such explanation—there is little reason to cite the presence of insiders as a key factor.

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European Soccer Leagues in the Dataset

Belgian First Division
German First Division
German Second Division
English Premier League
English Championship
English League 1
English League 2
English Conference
France Ligue 1
France Ligue 2
Greece Super League
Italy Serie
Italy Serie B
Netherlands Eredivisie
Portugal Primeira Liga
Scotland Premier League
Scotland Championship
Scotland League 1
Scotland League 2
Spain La Liga
Spain La Liga 2
Turkey Super Lig